A Note on Poincaré's Principle and the Behaviour of Moving Bodies and Clocks

Torgny Sjödin
Philosophisches Seminar, Göttingen

Z. Naturforsch. **35a**, 997–1000 (1980); received November 17, 1979

Exploiting the wave properties of light, the isotropy of space, and the homogeneity of space and time, the functions describing the behaviour of moving bodies and clocks are derived from Poincaré's principle about the impossibility to detect absolute motion.

Introduction

The Lorentz-Larmor ether theory has often been criticized because of its "ad hoc" assumptions about the contraction of bodies and slowing down of clocks moving with respect to the physical vacuum ("ether")*. The object of this paper is to use Poincaré's principle about the impossibility to detect motion with respect to the ether ("absolute motion") in order to derive the functions describing the behaviour of moving bodies and clocks. Poincaré probably began considering this principle in about 1895 [2]. It has met renewed interest recently, especially in Podlaha's work [3]. For other recent results in ether theory cf. also Refs. [4] and [5].

Besides Poincaré's principle we shall assume homogeneity of space and time, isotropy of space, and the existence of a wave in the ether (in the following called "light") travelling at the velocity c in all directions. As we shall see, given these assumptions, a very simple argument suffices to arrive at the conclusion that moving bodies undergo no transversal change of dimension, but a contraction in the direction of motion by the factor $\sqrt{1-w^2/c^2}$, and moving clocks are slowed down by the same factor.

Reprint requests to T. Sjödin, Istituto di matematica "Tonelli", via Buonarroti 2, I-56100 Pisa, Italy.

* We may here cite von Laue's book of 1911 [1]: "Eine eigentliche experimentelle Entscheidung zwischen der erweiterten Lorentzschen und der Relativitätstheorie ist dagegen wohl überhaupt nicht zu erbringen, und wenn die erstere trotzdem in den Hintergrund getreten ist, so liegt dies hauptsächlich daran, daß ihr, so nahe sie auch der Relativitätstheorie kommt, doch das große, einfache, allgemeine Prinzip mangelt, dessen Besitz der Relativitätstheorie schon in ihrer jetzigen, noch sehr der weiteren Entwicklung bedürftigen Gestalt etwas Imposantes verleiht."

Argument

Before commencing the deduction, let us explicitely state our decisive assumption, namely *Poincaré's principle:* It is impossible to detect absolute motion. This may be understood as: there is no way for an inertial observer to ascertain if he is at rest with respect to the ether, or not.

If a principle says that something is impossible, the best way to exploit it is usually to try to do the impossible and see which restrictions are then imposed by the principle. This is also the way to be followed in this paper. We shall consequently do our best to design a method to measure "absolute" velocities, i. e. velocities with respect to the privileged frame ("ether"). It will be found that – given the presuppositions stated in the introduction – Poincaré's principle completely determines the behaviour of (inertially) moving bodies and clocks.

In order to simplify, we shall in the following only consider a two-dimensional space. It is easily seen that this means no essential loss of generality.

We assume quite generally that the dimensions of bodies moving with respect to the ether are changed: in the direction of motion by the factor Φ, in the transversal direction by the factor Ψ, and that the rates of moving clocks are changed by the factor Ω. We assume that these functions are independent of time and space coordinates ("homogeneity of time and space") and are direction independent ("isotropy of space"). Since we shall here never consider accelerating objects, there is no need to make any assumptions about the possible acceleration dependence of the functions Φ, Ψ, and Ω. We only make the general assumption that the dimensions of a body, as well as the rate of a clock, only depend on its actual state of motion and not on its past. The functions may then be written: $\Phi = \Phi(|\underline{w}|)$, $\Psi =$

0340-4811 / 80 / 1000-0997 $ 01.00/0. — Please order a reprint rather than making your own copy.

$\Psi(|\underline{w}|)$, $\Omega = \Omega(|\underline{w}|)$. They are assumed to be differentiable.

Before being able to relate observations in different inertial systems, we must choose units and synchronization in these in some appropriate way. Since Poincaré's principle is assumed to be valid, methods of synchronization and of units stipulation must be chosen, whose realization is not dependent on the knowledge of absolute velocities. Taking this into consideration, we choose units in different inertial systems *preserving matter-geometry* [6] i.e. we assume that a rod which, when at rest in the privileged system S_0, has the length L as measured in S_0, will, after being accelerated and brought to rest in another inertial system S, have the same length when measured in S. Likewise for a clock. As concerns synchronization, we choose *natural synchronization* [5], i.e. we demand that a clock synchronized in one place and then moved with infinitesimal velocity ** (as measured in the system in question) to another place will also there show the "right" time. Since Ω is assumed to be radial, it holds $\Omega'(0) = 0$, which implies [5] that natural and *absolute synchronization* *** coincide in S_0. In the presupposition that the velocity of light is direction independent in the ether absolute synchronization is of course assumed. That in our case – owing to $\Omega'(0) = 0$ – the velocity of light in the ether is direction independent also under natural synchronization is an important fact to be made use of in the following.

Having thus stabilized the units and the synchronization in S, it is easy to derive a formula for the velocity of light in S that is assumed to move at constant velocity u with respect to S_0. This was already accomplished elsewhere [5]. If we denote the one-way velocity of light in S with $c'(\theta)$, where θ is the angle between the light ray and the direction of motion as measured in S, and the two-way velocity with $\bar{c}'(\theta)$, we obtain

$$c'(\theta) = ([1 - (1 - \Psi^2 \Phi^{-2} \gamma^{-2}) \sin^2 \theta]^{\frac{1}{2}} + [u/c + c\gamma^{-2} \Omega^{-1} \Omega'] \cos \theta)^{-1} c'_{\leftrightarrow}, \tag{1}$$

$$\bar{c}'(\theta) = [1 - (1 - \Psi^2 \Phi^{-2} \gamma^{-2}) \sin^2 \theta]^{-\frac{1}{2}} c'_{\leftrightarrow}, \tag{2}$$

where the two-way velocity along the direction of motion,

$$c'_{\leftrightarrow} = \bar{c}'(0),$$

is given by

$$c'_{\leftrightarrow} = \gamma^{-2} \Omega^{-1} \Phi^{-1} c. \tag{3}$$

Here c denotes the velocity of light in S_0, and γ is an abbreviation for $(1 - u^2/c^2)^{-\frac{1}{2}}$. The functions Φ, Ψ, and Ω are to be evaluated at u.

Let us now imagine an observer in some inertial system S, who wants to find out if he is moving with respect to the ether, or not. What would be more natural for him than to exploit the special properties of light for this purpose? – He knows that light is a signal, whose velocity in the ether is independent of the velocity of the source, and that it spreads isotropically in S_0; would he then through measurements find out that it has not this latter property with respect to S, he would immediately understand that he is moving with respect to the ether. We may hence conclude that *Poincaré's principle* implies:

(a) $\bar{c}'(\theta)$ is independent of θ,

(b) $c'(\theta) = \bar{c}'(\theta)$ for all θ.

Since this must hold for all inertial systems, we may from (a) and (2) directly conclude that

$$\Psi = \gamma \Phi, \tag{4}$$

and from (b) and (1) obtain the differential equation

$$1 + c^2 u^{-1} \gamma^{-2} \Omega^{-1} \Omega' = 0.$$

Taking into account that $\Omega(0) = 1$, we obtain as a solution to this equation:

$$\Omega = \gamma^{-1}. \tag{5}$$

It only remains to determine the function Ψ. To do this, we shall consider the function for the velocity of light in different systems, that we now may write

$$c'(u) = \Psi^{-1}(u) c. \tag{6}$$

Since Ψ is a radial function of u, c' must also be radial. Let us imagine that measurements of c' are

** Deciding if a velocity is small, or not, before having agreed upon the synchronization may look cumbersome. There are, however, at least two ways of solving this problem: 1) Using the concept of "self-measured velocity", as did Ives [7] and Bridgman [8]. 2) Introducing an auxiliary synchronization [9] employing light or any other signal.

*** For the realization of absolute synchronization the possibilities laying closest to hand are synchronization with faster than light signals or with Newtonian clocks that are not slowed down when moving with respect to the ether [5]. The facts that light is the fastest signal and that all clocks are Lorentzian, in no way deprive the *concepts* "signal faster than light" and "Newtonian clock" of their meaning; these are not logical but physical facts. Consequently, also the concept "absolute synchronization" is a meaningful concept even if de facto no physical operation exists to realize it.

made not only in S but in other inertial systems as well, moving with respect to S. We denote the resulting functions with $c_S'(v)$, where v denotes the relative velocity of the inertial systems as measured in S. Like before *Poincaré's principle* implies that also $c_S'(v)$ must be a radial function. In the case that u and v are parallel, we may take account of the transformation law for velocities [5]

$$u_2 = u_1 + (1 - \Phi(u_1)\Omega'(u_1)v)^{-1}\Phi(u_1)\Omega(u_1)v\,,$$

and from the equation $c_S'(v) = c_S'(-v)$ conclude that

$$c'(u + (1 - u\,v/c\,c_u')^{-1}\gamma^{-2}\,v\,c/c_u') = c'(u - (1 + u\,v/c\,c_u')^{-1}\gamma^{-2}\,v\,c/c_u'), \qquad (7)$$

where use was made of the Eqs. (4), (5), and (6) and c_u' stands for $c'(u)$.

The validity of Eq. (7) for every u and v now implies (this may be seen by differentiating both sides with respect to v and setting $v = 0$) that c' is a constant, i. e. $c'(u) \equiv c$, and hence $\Psi \equiv 1$ and $\Phi = \gamma^{-1}$. This completes our derivation.

Conclusion

We have seen that – given the presuppositions about isotropy of space, homogeneity of space and time, and light being a wave in the ether – there is only one world compatible with Poincaré's principle, namely the Lorentzian one. Some readers might still feel a little unsatisfied, though, thinking that the principal goal of an investigation of this kind should be the Lorentz transformations. As an answer to this we may quote Ives [10], who concludes "that the variations of mass, length, and frequency with motion are the primary physical phenomena, the Lorentz transformations are consequences". Assuming a choice of units preserving matter-geometry and natural synchronization, it is easy to derive the Lorentz transformations from the knowledge of the functions Φ, Ψ, and Ω [5, 11], but it is important to be aware of the fact that the same phenomena can also be described by means of other transformations; it is enough to choose units or synchronization in some other way [5, 12]. Likewise, different worlds may be described by the same transformations. Contrary to this, however, the correspondence worlds – sets of functions Φ, Ψ, Ω is one-to-one.

Let us finally, to the benefit of the reader not primarily interested in the ether, but interested in the matter from a more technical point of view, compare our derivation, seen as a derivation of the Lorentz transformations, with traditional derivations. The basic postulate of the traditional derivation of the Lorentz transformations is either the principle of relativity, demanding that all physical laws have the same form in all inertial systems, or the mathematical postulate that the transformations form a group. Besides the postulates of the homogeneity of space and time and the isotropy of space, very often the invariance of the velocity of light in all inertial systems is assumed [13]. It is true that there are many derivations without this latter postulate (The best of these is perhaps the one by Berzi and Gorini [14] that also contains an extensive bibliography. Cf. also [15]), but many of these make use of the so-called reciprocity principle, more or less explicitely. (For examples and a critic of this, see [14].) To the disadvantage of those few derivations that manage also without this principle (see [14] and references therein) it is to be said that they need to make use of rather complicated arguments.

In contrast to this, let us summarize the main virtues in technical respect of our derivation: 1) We did not assume the invariance of the velocity of light in different inertial systems, but only the *existence* of a wave in the ether; 2) We did not use the reciprocity principle; 3) The problem of the exclusion of an imaginary invariant velocity [14, 15] did not even arise; 4) Isotropy of space was only assumed in the privileged frame; 5) Not to be forgotten is that we managed without making the détour over the mathematical group postulate, and directly applied the pure physical postulate Poincaré's principle (and this only twice).

We see that even from the technical point of view our derivation offers its advantages in that it is a very simple derivation from few, physically well understood, postulates. Its main interest is though to be found in the fact that it is a deduction from a general principle making use of the ether concept. We hope that it will contribute to the eradication of the prejudice that the introduction of the ether complicates physical theory. On the contrary, the introduction of the concepts "privileged inertial system", "*real* changes of dimensions and of rates", and "absolute synchronization" – these being classical, physically easily understandable concepts – facilitates the understanding of physical theory. Still more important are of course the completely new avenues that open when one recognizes the need for a new fundamental theory to explain *why* bodies contract and clocks slow down when moving in the ether [4, 5, 16].

[1] M. von Laue, Das Relativitätsprinzip, § 3, Vieweg, Braunschweig 1911.
[2] G. H. Keswani, British Journal for the Philosophy of Science, **15**, 286 (1964).
[3] M. F. Podlaha, B. Althaus, and H. Baensch, Philosophia Naturalis, **16**, 315 (1977).
[4] M. F. Podlaha, Ind. J. Theor. Phys. **25**, 37 (1977); **26**, 181, (1978); **26**, 189 (1978); British Journal for the Philosophy of Science, **27**, 261 (1976); M. F. Podlaha and T. Sjödin, Lett. Nuovo Cim. **20**, 593 (1977).
[5] T. Sjödin, Nuovo Cim. **51 B**, 229 (1979).
[6] H. Reichenbach, Philosophie der Raum-Zeit-Lehre, de Gruyter, Berlin 1928.
[7] H. E. Ives: J. Opt. Soc. Amer. **29**, 294 (1939); **38**, 879 (1948); **39**, 757 (1949).
[8] P. W. Bridgman, A Sophisticate's Primer of Relativity, Wesleyan Univ. Press, Middletown, Conn. 1962.
[9] M. F. Podlaha, British J. Philosophy Sci. **30**, 376 (1979).
[10] H. E. Ives, Phil. Mag. **36**, 392 (1945).
[11] M. F. Podlaha and E. Navratil, Rev. Acad. Cienc. Exactas Madrid, **61**, 555 (1967).
[12] T. Sjödin, Eine Bemerkung zur physikalischen Realität und Koordinatentransformation, to be published.
[13] A. Einstein, Ann. Phys. **17**, 891 (1905).
[14] V. Berzi and V. Gorini, J. Math. Phys. **10**, 1518 (1969).
[15] G. Süssmann, Z. Naturforsch. **24 A**, 495 (1969).
[16] G. Cavalleri and Ø. Grøn, Lett. Nuovo Cim. **18**, 508 (1977).